

Magnetic-coupling-dependent spin-triplet supercurrents in helimagnet/ferromagnet Josephson junctions

Gábor B. Halász, M. G. Blamire, and J. W. A. Robinson*

*Department of Materials Science and Metallurgy, University of Cambridge,
Pembroke Street, Cambridge CB2 3QZ, United Kingdom*

The experimental achievements during the past year in demonstrating the existence of long-ranged spin-triplet supercurrents in ferromagnets proximity coupled to singlet superconductors open up the possibility for new interesting physics and applications [for a review, see M. Eschrig, *Phys. Today* **64**(1), 43 (2011)]. Our group reported the injection of triplet supercurrents into a magnetically uniform ferromagnet (Co) by sandwiching it between two helimagnet/superconductor (Ho/Nb) bilayers to form a Nb/Ho/Co/Ho/Nb-type Josephson device. In the function of the Ho layer thicknesses, the supercurrent was found to modulate in a complex way that seemed to depend on the magnetic structure of Ho. To understand this unusual behavior, we have theoretically studied the properties of an ideal Josephson device with a helimagnet/ferromagnet/helimagnet (HM/F/HM) barrier in the clean limit using the Eilenberger equation; we show, in particular, that the maximum triplet supercurrent that can pass across the barrier will depend non-monotonically on the thicknesses of the HM layers if the HM and F layers are magnetically exchange coupled at their interface.

I. INTRODUCTION

The intense interest in understanding the interplay between superconductors (S) and ferromagnets (F) was primarily triggered by the pioneering work of Ryazanov *et al.*,¹ who showed that the supercurrent in a S/F/S junction containing the weak ferromagnet CuNi was periodically modulated: first by temperature as a result of varying the exchange energy of the low Curie temperature CuNi, and also in the function of the CuNi thickness.² The oscillating and decay lengths of the thickness modulation are much shorter ($\xi \approx 1$ nm) if a strong ferromagnet is used, such as Co,^{3–5} Fe,^{6,7} or Ni,^{3,8} but because of their long mean free paths for both scattering and spin flip they are attractive materials to use in S/F/S junctions. For a detailed review on the singlet proximity coupling of S and F materials see Ref. 9 and references therein.

The decay lengths of supercurrents in ferromagnets can be radically extended if the electron pairs transform from a singlet to a triplet spin state at a S/F interface via a spin-mixing process. This is known as the long-range triplet proximity effect; for a comprehensive review, see Ref. 10 and references therein. One way to theoretically promote the spin-mixing conversion between singlet pairs and triplet pairs at a S/F interface is to incorporate a magnetically inhomogeneous layer between the S and F layers.¹¹ The triplet electron pairs which form are expected to be much more robust in ferromagnets, with longer coherence lengths ($\xi \gg 1$ nm) than pairs in a singlet state. Therefore the demonstration of a supercurrent in a S/F/S junction where the F layer thickness exceeds any length scale possible for a singlet pair to exist is considered indirect proof of the long-range triplet proximity effect. Signatures of such a long-range proximity effect were first reported more than two decades ago (see, e.g. Ref. 12). However, it was only in 2006 when the first major breakthrough was achieved with supercurrents reported in planar Josephson devices with half-metallic (i.e., fully spin polarized) barriers that were hundreds of nanometers long;¹³ see also the related articles in Ref. 14. In the same year, a triplet superconducting state was also reported in Ho, a helimagnetic rare-earth metal, which formed

the junction of a superconducting interferometer.¹⁵ These results stimulated intense theoretical work aimed at understanding triplet supercurrents better and how they could be created at S/F interfaces in a more routine way.

Over the past year it would seem that the puzzle to control triplet pair generation in S/F/S devices was finally solved with long-ranged supercurrents consistent with spin-triplet theory demonstrated in a wide range of ferromagnets,¹⁶ including a Co/Ru/Co synthetic antiferromagnet interfaced by normal (i.e., non-magnetic) metal spacers and ferromagnetic alloys¹⁷ (see also Ref. 18); a Ho/Co/Ho composite barrier¹⁹ (see also Refs. 20–22); a magnetic Cu₂MnAl Heusler compound with a complex magnetic profile²³ (see also Ref. 24); a half-metallic CrO₂ wire;²⁵ and a Co nanowire.²⁶ In addition to these experiments, superconducting gap features have been very recently measured by scanning tunneling spectroscopy²⁷ in the half-metallic manganite La_{2/3}Ca_{1/3}MnO₃ (LCMO) grown on YBa₂Cu₃O_{7- δ} . The gap features were observed in the LCMO up to a thickness of approximately 30 nm, and so offer the potential to explore the fundamental properties of a triplet superconducting state.

The experiments reported in Refs. 17, 19, and 23 share many similarities with the theoretical triplet junction proposed by Houzet and Buzdin:²⁸ a S/F'/F''/S junction in which the magnetizations in the F' and F'' layers are non-collinear with the magnetization in the F layer, allowing control over the creation of triplet electron pairs. Our group reported Josephson devices in which the F' and F'' layers were substituted by Ho and the F layer used was Co.¹⁹ The maximum supercurrent (i.e., the Josephson critical current I_C) in these devices was found to depend non-monotonically on the thickness d_h of each Ho layer, with peaks in I_C when $d_h \approx 4.5$ nm and $d_h \approx 10$ nm. These thicknesses appeared to correspond to an optimum spin-triplet proximity effect, and by increasing the Co thickness d_f in the 0–16 nm range, a slow decay in I_C was observed with a decay length of ~ 10 nm at 4.2 K. This decay length is almost ten times larger than that found in simple (spin-singlet dominated) Co-based Josephson devices.^{3,4} Alidoust and Linder²⁰ established theoretically that

spin-triplet supercurrents could account for the slow decay in I_C with d_f , but their model did not account for the complex dependence of I_C on d_h . When rare-earth ferromagnets are grown on transition-metal ferromagnets, it is well known that the two materials magnetically couple at the interface. In view of this fact, we solve the Eilenberger equation for an ideal S/HM/F/HM/S Josephson device with arbitrary interfacial magnetic coupling at the HM/F interfaces, and demonstrate a profound dependence of the triplet supercurrent amplitude on the thicknesses of the HM layers. The results provide a new level of understanding into the conversion process between singlet and triplet Cooper pairing at a superconductor/helomagnet/ferromagnet interface.

II. GENERAL FORMALISM

The S/HM/F/HM/S device considered in this paper is illustrated in Fig. 1. All layers are normal to the z direction. The HM layers have a thickness d_h and their magnetization rotates in the $\{x, y\}$ plane as a function of z .³¹ The strongly ferromagnetic F layer has a thickness d_f and a uniform magnetization in the $+x$ direction. The S layers at $z < 0$ and $z > d \equiv 2d_h + d_f$ are infinitely thick and in a singlet state. Subscripts h , f , and s refer to the HM, F, and S layers.

We assume a moderately clean limit and a temperature close to the critical temperature T_C of the superconducting leads, therefore we adapt the linearized Eilenberger equation (LEE). Due to the presence of non-uniform magnetization, we consider both singlet and triplet correlations. The anomalous Green's function is then a 2×2 matrix given by $\hat{f}(z, \theta, \omega) = f^0 \hat{1} + \vec{f} \cdot \vec{\sigma}$, where $\vec{f} = (f^x, f^y, f^z)$, and $\vec{\sigma} = (\hat{\sigma}^x, \hat{\sigma}^y, \hat{\sigma}^z)$ is a vector of the Pauli matrices. The component f^0 is associated with spin-singlet pairs, while the three components in \vec{f} are associated with spin-triplet pairs. To understand the meaning of the different triplet components, we switch to an alternative representation where we have $\hat{f} = f^0 \hat{1} + f^x \hat{\sigma}^x + f^+ \hat{\sigma}^+ + f^- \hat{\sigma}^-$ with $\hat{\sigma}^\pm = (\hat{\sigma}^y \pm i \hat{\sigma}^z)/\sqrt{2}$ and $f^\pm = (f^y \mp i f^z)/\sqrt{2}$. The components f^x and f^\pm then correspond to triplet pairs with spin projections 0 and ± 1 to the z direction, respectively.

Since the HM layers have a weak magnetization, all components in \hat{f} are non-zero. The Fermi surface average is $\langle \hat{f} \rangle \approx 0$ in the moderately clean limit²⁹ and so the LEE reads

$$v_h \cos \theta \frac{\partial \hat{f}_h}{\partial z} + (2\omega + \tau_h^{-1}) \hat{f}_h + i \vec{I}_h \cdot \{\vec{\sigma}, \hat{f}_h\} = 0, \quad (1)$$

where $\{x, y\} \equiv xy + yx$ is the anticommutator, while v_a and τ_a are the Fermi velocity and the pair-breaking time in a generic layer a . The electron mean free path is then $\ell_a = v_a \tau_a$. The Matsubara frequencies are given by $\omega = \pi k_B T (1 + 2n)/\hbar$, and \vec{I}_h is the magnetic exchange field in units of frequency. This field takes the form $\vec{I}_h = I_h (\cos[Q\tilde{z} + \alpha], \sin[Q\tilde{z} + \alpha], 0)$ in the HM layers, where $\tilde{z} \equiv |z - d/2| - d_f/2$ is the distance to the nearest HM/F interface and Q is the wave vector of the magnetic helix. The magnetic exchange coupling at the HM/F interfaces is included by

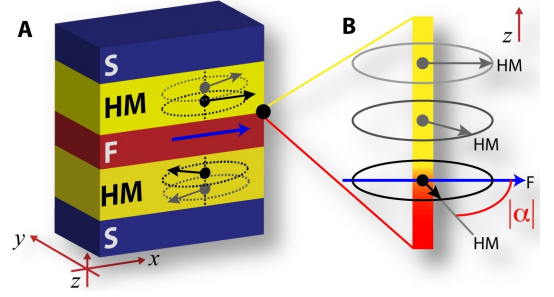


FIG. 1: (Color online) (A) Illustration of a superconducting Josephson device with a helimagnet/ferromagnet/helomagnet barrier (S/HM/F/HM/S). The HM layers control the conversion between singlet and triplet Cooper pairing, while the F layer behaves like a spin filter, only allowing triplet Cooper pairs to pass across it. The F and HM layers are magnetically exchange coupled at the F/HM interfaces (B) with an angle of $-\pi \leq \alpha \leq \pi$ radians between the F and HM layer moments ($\alpha < 0$ in the figure).

introducing an anisotropy angle α between the local magnetizations of the HM and F layers. When α is 0 or π , the magnetizations are locally parallel or antiparallel, respectively.³²

For the strongly magnetized F layer there are two independent spin bands for the majority ($+x$) and the minority ($-x$) spins, and the singlet correlations are destroyed on a sub-nanometer scale. Therefore we take $f^0 = f^x = 0$ and identify the remaining components f^\pm with triplet pairs from the majority and minority spin bands. The governing equations for the two independent spin bands are then

$$v_f^\pm \cos \theta \frac{\partial f_f^\pm}{\partial z} + (2\omega + \tau_f^{-1}) f_f^\pm = 0, \quad (2)$$

where we assume $v_f^+ > v_f^-$. However, since scattering is mainly due to impurities at low temperatures, we assume that the pair-breaking time τ_f is the same for both spin bands.

At the interface $z = z_{ab}$ between two generic layers a and b , the parallel component of the Fermi momentum is conserved. If we assume that all bands are parabolic with the same effective electron mass, the Fermi momentum is proportional to the Fermi velocity, meaning $\hat{f}_a(z_{ab}, \theta_a, \omega) = \hat{f}_b(z_{ab}, \theta_b, \omega)$ for $v_a \sin \theta_a = v_b \sin \theta_b$. If we further assume that $v_s \ll v_h, v_f$, it is valid to use the single-channel approximation; the anomalous Green's function \hat{f} is then only non-zero for angles close to 0 or π in the HM and F layers, therefore $\cos \theta$ can be substituted by ± 1 in Eqs. (1) and (2).

The phase difference between the two S layers is Φ and the magnitude Δ of the bulk pairing potentials (Δ and $\Delta e^{i\Phi}$) is determined from the self-consistency equation. For $T \approx T_C$, we can apply rigid boundary conditions at the S/HM interfaces.²⁸ This means that \hat{f} is the same at the interface as in the bulk of the S layers for outgoing directions. The boundary conditions at the S/HM interfaces are then $\hat{f}_s(0, \theta, \omega) = F(\Delta, \omega) \hat{1}$ for $\cos \theta > 0$ and $\hat{f}_s(d, \theta, \omega) = F(\Delta, \omega) e^{i\Phi} \hat{1}$ for $\cos \theta < 0$, where $F(\Delta, \omega) = \Delta / \sqrt{\Delta^2 + \hbar^2 \omega^2}$ is the equilibrium value of f^0 and the subscript s indicates that the expressions correspond to the S side of the interface.

In the formalism of the LEE, the Josephson current density at a generic position z in the junction is given by

$$J = \frac{3\pi k_B T \sigma_z}{2e\ell_z} \sum_{\omega>0} \int \frac{d\Omega}{4\pi} \cos \theta \operatorname{Im} \operatorname{Tr} \left[\hat{f}^\dagger(z, \theta, \omega) \hat{f}'(z, \theta', \omega) \right], \quad (3)$$

where $\theta' \equiv \pi - \theta$, and σ_z is the normal-state conductivity of the layer at position z . The function \hat{f}' satisfies the same equations as \hat{f} with the same boundary conditions at the interfaces, reversed exchange field ($-\vec{I}_h$) in the HM layers, and exchanged spin bands ($v_f^+ \leftrightarrow v_f^-$) in the F layer.

III. CRITICAL CURRENT OF THE JUNCTION

In this section we calculate the Josephson current at the S side of the $z = d$ interface and hence obtain the critical current of the junction. Since $\hat{f}_s(d, \theta, \omega)$ for $\cos \theta < 0$ is known and only its f^0 component is non-zero, the Josephson current is determined entirely by $f_s^0(d, \theta, \omega)$ for $\cos \theta > 0$. Due to the linearity of Eqs. (1) and (2), we can write

$$f_s^0(d, \theta, \omega) = S(\theta, \omega) f_s^0(0, \theta, \omega) = S(\theta, \omega) F(\Delta, \omega), \quad (4)$$

$$S(\theta, \omega) = \mathbf{S}_h^T \cdot \mathbf{S}_f \cdot \mathbf{S}_h, \quad (5)$$

where the different matrices/vectors \mathbf{S} describe the effects of the different layers between $z = 0$ and $z = d$.

The role played by the HM layers is to convert between the singlet component f^0 in the S layers and the triplet components f^\pm in the F layer, implying \mathbf{S}_h is a 2×1 vector, while \mathbf{S}_h^T is a 1×2 vector. The expressions for \mathbf{S}_h and \mathbf{S}_h^T can be obtained by solving Eq. (1) in the two HM layers. The vector corresponding to the lower HM is

$$\mathbf{S}_h = -iB \exp(-D_h) \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad (6)$$

$$B = \frac{q \cos \alpha (1 - \cos \delta_h) + \sqrt{1 + q^2} \sin \alpha \sin \delta_h}{\sqrt{2}(1 + q^2)}, \quad (7)$$

where $q \equiv Q\xi_h$, and $\xi_h = v_h/2I_h$ is the correlation oscillating length in the HM layers. We also define reduced thicknesses as $\delta_h \equiv d_h \sqrt{1 + q^2}/\xi_h$ and $D_a \equiv d_a(2\omega + \tau_a^{-1})/v_a$, where the latter one is valid for a generic layer a . Physically, the parameter B describes the conversion efficiency between singlet and triplet Cooper pairs in each HM layer, while B^2 describes the total efficiency of both HM layers.

Since the non-zero triplet components f^\pm are independent in the F layer, \mathbf{S}_f is a 2×2 diagonal matrix. Integrating Eq. (2) gives

$$\mathbf{S}_f = \begin{pmatrix} \exp(-D_f^+) & 0 \\ 0 & \exp(-D_f^-) \end{pmatrix}. \quad (8)$$

The fact that $v_f^- < v_f^+$ implies $D_f^- > D_f^+$, therefore the triplet component f^- corresponding to the minority spin band decays faster than f^+ .

Equations (6)–(8) were derived by setting $\cos \theta \approx 1$ in the HM and F layers. Consequently, $S(\omega) \equiv S(\theta, \omega)$ is independent of θ . If we substitute Eq. (4) and $\hat{f}_s(d, \theta, \omega) = F(\Delta, \omega) e^{i\Phi} \hat{1}$ (for $\cos \theta < 0$) into Eq. (3), we recover the usual current-phase relation, $J = J_C \sin \Phi$, and the critical current density becomes

$$J_C = \frac{3\pi k_B T \sigma_s}{e\ell_s} \int_0^1 d\zeta \cdot \zeta \sum_{\omega>0} F(\Delta, \omega)^2 \operatorname{Re}[S(\omega)]. \quad (9)$$

The integral in $\zeta = \cos \theta$ gives $1/2$, while the sum in ω requires a further approximation. Since $\omega \ll \tau_a^{-1}$ for all layers and all Matsubara frequencies with a significant contribution, we can neglect ω next to τ_a^{-1} in D_a . This implies that $D_a \approx d_a/\ell_a$ and that $S \equiv S(\omega)$ is independent of ω as well. The sum is now evaluated using $\sum_{n=0}^{\infty} \frac{1}{p^2 + (1+2n)^2} = \frac{\pi}{4p} \tanh\left(\frac{p\pi}{2}\right)$ and so the characteristic voltage reads

$$I_C R_N = \frac{3\pi \Delta \sigma_s}{8e\ell_s} \tanh\left(\frac{\Delta}{2k_B T}\right) \left(\frac{2d_h}{\sigma_h} + \frac{d_f}{\sigma_f}\right) S, \quad (10)$$

$$S = -B^2 \exp\left(-\frac{2d_h}{\ell_h}\right) \sum_{\pm} \exp\left(-\frac{d_f}{\ell_f^{\pm}}\right). \quad (11)$$

The two terms in the sum are contributions from triplet pairs traveling through the majority and minority spin bands of the F layer, respectively.

IV. DISCUSSION

Since only triplet pairs can enter the strongly ferromagnetic F layer, the conversion between singlet and triplet pairs in the HM layers is crucial. This process is represented by the efficiency parameter B^2 in Eq. (11) and in this section we discuss how the conversion depends on d_h , q , and α .

We first note from Eq. (7) that B^2 is π periodic in the anisotropy angle α . This implies that the critical current does not distinguish between locally parallel and antiparallel magnetizations of the HM and F layers. In both of these cases, the efficiency parameter takes the form

$$B^2 = \frac{2q^2}{(1 + q^2)^2} \sin^4\left(\frac{d_h \sqrt{1 + q^2}}{2\xi_h}\right) \quad (\alpha = 0, \pi), \quad (12)$$

which shows that the periodicity of B^2 in d_h is in general $\Delta d_h = 2\pi\xi_h/\sqrt{1 + q^2}$. The periodicity Δd_h depends on the two intrinsic length scales of the HM layers: the oscillating length ξ_h and the wavelength $\lambda = 2\pi/Q$ of the magnetic helix. Furthermore, Eq. (12) shows that the shorter length scale is the dominant one. When the oscillating length is small (i.e., $q \ll 1$), we have $\Delta d_h \approx 2\pi\xi_h$ and so the helical structure becomes irrelevant. The periodicity is determined by ξ_h as for uniform magnets. When the oscillating length is large (i.e., $q \gg 1$), we have $\Delta d_h \approx \lambda$ and so that the periodicity is determined entirely by the helical structure. It also follows from

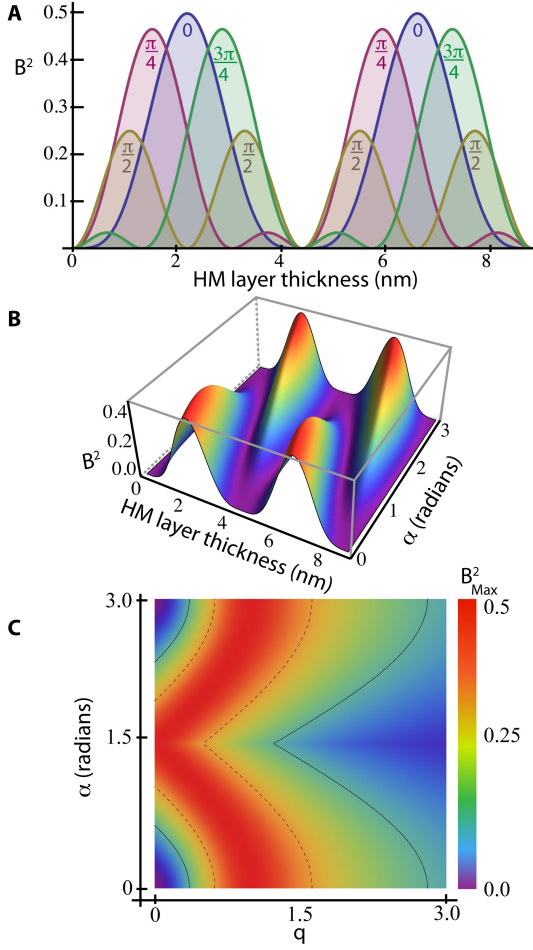


FIG. 2: (Color online) The efficiency parameter B^2 peaks at particular HM layer thicknesses (A), and the positions and magnitudes of these peaks depend on the magnetic anisotropy angle α between neighboring HM and F layer moments (A, B). In (A) and (B), $q = 1$. For optimum HM layer thicknesses, the maximum efficiency parameter B^2_{max} depends sensitively on q and α (C).

Eq. (12) that the zeros of B^2 are at $d_h = m \Delta d_h$, and that its maxima are halfway between its zeros.

If we start increasing α , the periodicity Δd_h remains unchanged, and B^2 still has zeros at $d_h = m \Delta d_h$. However, the primary maxima halfway are shifted to the left, and secondary maxima appear with additional zeros in between (see Fig. 2). When the local HM and F magnetizations are orthogonal, the efficiency parameter reads

$$B^2 = \frac{1}{2(1+q^2)} \sin^2 \left(\frac{d_h \sqrt{1+q^2}}{\xi_h} \right) \quad (\alpha = \pm\pi/2). \quad (13)$$

The primary and secondary maxima become equivalent, the periodicity in d_h is reduced by 2, and the zeros of B^2 occur at $d_h = m \Delta d_h/2$.

It is interesting to look at the efficiency of the singlet/triplet conversion at optimal HM layer thicknesses. This is determined by q and α . In the limit when $q \gg 1$, the critical current

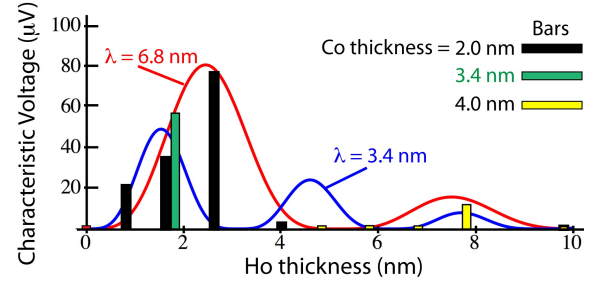


FIG. 3: (Color online) Experimental (bars, adapted from Ref. 19) and theoretical (curves) $I_C R_N$ values of a Nb/Ho/Co/Ho/Nb device against the symmetrical thickness of Ho at 4.2 K. Theoretical curves are plotted for $\lambda = 3.4$ nm (blue) and $\lambda = 6.8$ nm (red), while the Co thickness is kept at 3.4 nm (note that the local magnitude of $I_C R_N$ only weakly depends on the Co thickness). For the experimental $I_C R_N$ values, the magnetically dead layers present at each Ho surface (≈ 1.2 nm) have been subtracted from the total Ho thickness. Theoretical curves are plotted with the following realistic parameter values: $T_C = 9.1$ K, $v_s = 0.4 \times 10^6$ ms $^{-1}$, $v_h = v_f^+ = 1.0 \times 10^6$ ms $^{-1}$, $v_f^- = 0.75 \times 10^6$ ms $^{-1}$, $\ell_h = 4$ nm, $\ell_f^+ = 12$ nm, $\ell_f^- = 9$ nm, $I_h = 0.25$ eV, and $\alpha = 0$.

vanishes. This is because if λ is too small, the HM magnetization averages to zero within the length scale of ξ_h . In the opposite limit when $q \ll 1$, we need $\alpha \neq 0$ for $I_C \neq 0$. If λ is too large, the HM layers become uniformly magnetized, and we recover the results in Ref. 28 as $I_C \propto B^2 \propto \sin^2 \alpha$. To be more quantitative, we maximize B^2 with respect to d_h ; this calculation gives

$$B^2_{\text{max}} = \frac{(q |\cos \alpha| + \sqrt{q^2 + \sin^2 \alpha})^2}{2(1+q^2)^2}, \quad (14)$$

and the dependence of B^2_{max} on q and α is shown in Fig. 2(C). We can establish that the most efficient singlet/triplet conversion with $B^2_{\text{max}} = 1/2$ occurs whenever $q = |\cos \alpha|$. This is in fact an absolute theoretical maximum. Due to the two triplet channels f^\pm in the F layer, $B^2_{\text{max}} = 1/2$ corresponds to perfect conversion.

Altogether, the dependence of B^2 on the HM layer thickness shown in this paper demonstrates the profound effect that interfacial layer-by-layer magnetic coupling can have on the amplitude of the triplet supercurrent in a S/HM/F/HM/S device. Even when the magnetizations at the HM/F interfaces couple parallel or antiparallel, the amplitude of the triplet supercurrent oscillates with peaks and zeros commensurate on d_h . This result agrees with experimental I_C data reported in Ref. 19: in Fig. 3 we directly compare the theoretical and experimental dependence of $I_C R_N$ on Ho layer thickness and agreement is achieved for λ in the 3.4–6.8 nm range, consistent with λ values estimated in Ho thin films.³⁰

This close agreement between theory and experiment provides a clearer understanding of the role played by interfacial magnetic coupling between Ho and Co on the I_C found in Nb/Ho/Co/Ho/Nb devices.¹⁹ The possibility of controlling I_C by manipulating the coupling anisotropy (i.e., α) in these or

similar devices has not been explored experimentally so far. Since the Curie temperature of Co (~ 1000 K) is significantly higher than either the Néel (~ 130 K) or the Curie (~ 20 K) temperature of Ho, it may be possible to vary α by field cooling a device from ~ 130 K. By repeating this procedure with the field applied at various in-plane angles, the effect of α on the amplitude of the triplet supercurrent I_C could be tested with potentially large $\Delta I_C/I_C$ ratios obtainable.

V. SUMMARY

Layer-by-layer interfacial magnetic coupling between rare-earth helimagnets (HM) and ferromagnets (F) can strongly in-

fluence the interconversion between singlet and triplet Cooper pairing in S/HM/F/HM/S-type Josephson devices. This results in the amplitude of the spin-triplet supercurrent passing through the F layer depending sensitively on the magnetic structure and the thicknesses of the helimagnetic layers.

Acknowledgments

We are grateful to F. S. Bergeret, A. I. Buzdin, F. Chiodi, and J. Linder for valuable advice during the preparation of this paper. The research was funded by St. John's College, Cambridge, and the UK EPSRC.

-
- * Electronic address: jjr33@cam.ac.uk
- ¹ V. V. Ryazanov, V. A. Oboznov, A. Yu. Rusanov, A. V. Veretenikov, A. A. Golubov, and J. Aarts, Phys. Rev. Lett. **86**, 2427 (2001).
 - ² V. A. Oboznov, V. V. Bol'ginov, A. K. Feofanov, V. V. Ryazanov, and A. I. Buzdin, Phys. Rev. Lett. **96**, 197003 (2006).
 - ³ J. W. A. Robinson, S. Piano, G. Burnell, C. Bell, and M. G. Blamire, Phys. Rev. Lett. **97**, 177003 (2006); Phys. Rev. B **76**, 094522 (2007).
 - ⁴ J. W. A. Robinson, Z. H. Barber, and M. G. Blamire, Appl. Phys. Lett. **95**, 192509 (2009).
 - ⁵ M. A. Khasawneh, W. P. Pratt, Jr., and N. O. Birge, Phys. Rev. B **80**, 020506(R) (2009).
 - ⁶ S. Piano, J. W. A. Robinson, G. Burnell, and M. G. Blamire, Eur. Phys. J. B **58**, 123 (2007).
 - ⁷ J. W. A. Robinson, G. B. Halász, A. I. Buzdin, and M. G. Blamire, Phys. Rev. Lett. **104**, 207001 (2010).
 - ⁸ A. A. Bannykh, J. Pfeiffer, V. S. Stolyarov, I. E. Batov, V. V. Ryazanov, and M. Weides, Phys. Rev. B **79**, 054501 (2009).
 - ⁹ A. I. Buzdin, Rev. Mod. Phys. **77**, 935 (2005).
 - ¹⁰ F. S. Bergeret, A. F. Volkov, and K. B. Efetov, Rev. Mod. Phys. **77**, 1321 (2005).
 - ¹¹ F. S. Bergeret, A. F. Volkov, and K. B. Efetov, Phys. Rev. Lett. **86**, 4096 (2001).
 - ¹² M. D. Lawrence and N. Giordano, J. Phys. Condens. Matter **8**, L563 (1996); M. Giroud, H. Courtois, K. Hasselbach, D. Mailly, and B. Pannetier, Phys. Rev. B **58**, 11872(R) (1998); V. T. Petrashov, I. A. Sosnin, I. Cox, A. Parsons, and C. Troadec, Phys. Rev. Lett. **83**, 3281 (1999).
 - ¹³ R. S. Keizer, S. T. B. Goennenwein, T. M. Klapwijk, G. Miao, G. Xiao, and A. Gupta, Nature (London) **439**, 825 (2006).
 - ¹⁴ M. Eschrig, J. Kopu, J. C. Cuevas, and G. Schön, Phys. Rev. Lett. **90**, 137003 (2003); M. Eschrig and T. Löfwander, Nat. Phys. **4**, 138 (2008).
 - ¹⁵ I. Sosnin, H. Cho, V. T. Petrashov, and A. F. Volkov, Phys. Rev. Lett. **96**, 157002 (2006).
 - ¹⁶ For an overview, see M. Eschrig, Phys. Today **64**(1), 43 (2011).
 - ¹⁷ T. S. Khaire, M. A. Khasawneh, W. P. Pratt, Jr., and N. O. Birge, Phys. Rev. Lett. **104**, 137002 (2010).
 - ¹⁸ L. Trifunovic and Z. Radović, Phys. Rev. B **82**, 020505(R) (2010); A. F. Volkov and K. B. Efetov, Phys. Rev. B **81**, 144522 (2010).
 - ¹⁹ J. W. A. Robinson, J. D. S. Witt, and M. G. Blamire, Science **329**, 59 (2010).
 - ²⁰ M. Alidoust and J. Linder, Phys. Rev. B **82**, 224504 (2010).
 - ²¹ G. B. Halász, J. W. A. Robinson, J. F. Annett, and M. G. Blamire, Phys. Rev. B **79**, 224505 (2009).
 - ²² I. T. M. Usman, K. A. Yates, J. D. Moore, K. Morrison, V. K. Pecharsky, K. A. Gschneidner, T. Verhagen, J. Aarts, V. I. Zverev, J. W. A. Robinson, J. D. S. Witt, M. G. Blamire, and L. F. Cohen, Phys. Rev. B **83**, 144518 (2011).
 - ²³ D. Sprungmann, K. Westerholt, H. Zabel, M. Weides, and H. Kohlstedt, Phys. Rev. B **82**, 060505(R) (2010).
 - ²⁴ J. Linder and A. Sudbø, Phys. Rev. B **82**, 020512(R) (2010).
 - ²⁵ M. S. Anwar, F. Czeschka, M. Hesselberth, M. Porcu, and J. Aarts, Phys. Rev. B **82**, 100501(R) (2010).
 - ²⁶ J. Wang, M. Singh, M. Tian, N. Kumar, B. Liu, C. Shi, J. K. Jain, N. Samarth, T. E. Mallouk, and M. H. W. Chan, Nat. Phys. **6**, 389 (2010).
 - ²⁷ Y. Kalcheim, T. Kirzhner, G. Koren, and O. Millo, Phys. Rev. B **83**, 064510 (2011).
 - ²⁸ M. Houzet and A. I. Buzdin, Phys. Rev. B **76**, 060504(R) (2007).
 - ²⁹ F. Konschelle, J. Cayssol, and A. I. Buzdin, Phys. Rev. B **78**, 134505 (2008).
 - ³⁰ V. Leiner, D. Laberge, R. Siebrecht, C. Sutter, and H. Zabel, Physica B **283**, 167 (2000); L. He, Solid State Commun. **151**, 651 (2011); C. Bryn-Jacobsen, R. A. Cowley, D. F. McMorro, J. P. Goff, R. C. C. Ward, and M. R. Wells, Phys. Rev. B **55**, 317 (1997).
 - ³¹ Here we neglect the constant axial magnetization that is present in crystalline Ho below 20K to simplify the mathematics; nevertheless, the same conclusions are achieved if we include this component.
 - ³² Note that when $\alpha < 0$ the HM magnetization rotates too much as $\tilde{z} \rightarrow 0$ and when $\alpha > 0$ it does not rotate enough. The symmetry in the definition of \tilde{z} assumes that the helicities of the two HM layers are opposite. It turns out that reversing the helicity of one layer only gives an overall minus sign in the expression for the critical current, which is not measurable.